$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}$ find the 77^{th} term of the sequence

Sol^n:- Reciprocal of the Numbers are $2, 6, 12, 20, \text{-----}$ → Original Sequence.

We will use Method of finite difference to find n^{th} term.

2,	6,	12,	20 ------	Original Sequence.
4	6	8	-- ----	First difference between 2 consecutive Numbers.
2	2	2	- ----	2nd difference between 2 consecutive Numbers.

So n^{th} term will be of form $T_n = an^2 + bn + c$

Let $n=1$ $\quad T_1 = a(1) + b(1) + c = 2$ ——— ①

$n=2$ $\quad T_2 = a(2)^2 + b(2) + c = 6 \Rightarrow 4a + 2b + c = 6$ ——— ②

$n=3$ $\quad T_3 = a(9) + b(3) + c = 12$ ——— ③

Eq ② − ① we get $3a + b = 4$ $\quad$ So $3(1) + b = 4$

③ − ② we get $5a + b = 6$ $\quad b = 1$

$-2a = -2$

$a = 1$

Now from ① $\quad a + b + c = 2$

$1 + 1 + c = 2 \quad c = 0$

So $T_n = an^2 + bn + c = n^2 + n = n(n+1)$

$T_{77} = 77(77+1) = 77 \times 78 = 6006$

So 77^{th} term in Sequence is $\frac{1}{6006}$

Alternate solution.

$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}$ -----

$\frac{1}{1\times2}, \frac{1}{2\times3}, \frac{1}{3\times4}, \frac{1}{4\times5}$ ------

So n^{th} term is $\frac{1}{n(n+1)}$

$T_{77} = \frac{1}{77\times78} = \frac{1}{6006}$