Find $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$

Sol^n:- $\frac{1}{n^2+3n+2} = \frac{1}{(n+1)(n+2)}$

We can resolve $\frac{1}{(n+1)(n+2)} = \frac{A}{(n+1)} + \frac{B}{(n+2)}$

$$\frac{1}{(n+1)(n+2)} = \frac{A(n+2) + B(n+1)}{(n+1)(n+2)}$$

So $1 = A(n+2) + B(n+1)$

Let $n = -1$ $\quad 1 = A(-1+2) + B(0)$ or $A = 1$

Let $n = -2$ $\quad 1 = A(-2+2) + B(-2+1)$ or $B = -1$

So $\frac{1}{(n+1)(n+2)} = \frac{A}{(n+1)} + \frac{B}{(n+2)} = \frac{1}{(n+1)} - \frac{1}{(n+2)}$

Now $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2} = \sum_{n=1}^{\infty} \frac{1}{n+1} - \sum^{\infty} \frac{1}{(n+2}$

$$= \frac{1}{2} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} \cdots - \left(\cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} + \cdots\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2} = \frac{1}{2}$$