Two numbers are selected at random from the integers 1 to 9. If the sum of the two numbers is even, find the probability that both numbers are odd?

Sol^{n} :- A = event that Sum of two Numbers is even.

B = event that both Numbers are odd.

We have to find conditional Probability $P(B|A)$

We know $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(B)$ = Probability of selecting two Number which are odd = $\frac{5C_2}{9C_2}$

$P(A)$ = Probability of sum of 2 Numbers is even = $\frac{(5C_2 + 4C_2)}{9C_2}$

[∵ There are 5 odd Numbers & 4 even Numbers
Sum of 2 odd Numbers is even & Sum of 2 even Numbers is even]

$P(B \cap A) = \frac{5C_2}{9C_2}$ (∵ $A = [5C_2 + 4C_2]$. $B = 5C_2$ so $A \cap B = 5C_2$]

So $P(B|A)$ = Probability of selecting 2 odd Number so that Sum is even

$$= \frac{\frac{5C_2}{9C_2}}{\frac{(5C_2 + 4C_2)}{9C_2}} = \frac{5C_2}{5C_2 + 4C_2} = \frac{\frac{5!}{3!\,2!}}{\frac{5!}{3!\,2!} + \frac{4!}{2!\,2!}} = \frac{\frac{5\times4}{2}}{\frac{5\times4}{2} + \frac{4\times3}{2}}$$

$$= \frac{20}{20+12} = \frac{20}{32} = \frac{10}{16} = \frac{5}{8}$$

So $P(B|A) = \frac{5}{8}$