A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning ?

Soln.:- A = Event that A gets a Number > 4 & wins

So $P(A)$ = Probability of A winning $= \frac{2}{6} = \frac{1}{3}$

$P(\bar{A})$ = Probability of A losing $= 1 - 1/3 = 2/3$

B = Event that B gets a Number > 4 & wins. $P(B) = 1/3$

$P(\bar{B})$ = Probability of B Not winning $= 1 - 1/3 = 2/3$

B can win in 2nd, 4th, 6th ----- even Numbers of throws.

P(B winning in 2nd throw) $= P(B | \bar{A}) = \frac{1}{3} \times 2/3$

P(B winning on 4th throw) $= P(B | \bar{A} | \bar{B} | \bar{A}) = 1/3 \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3}(2/3)^3$

P(B winning on 6th throw) $= P(B | \bar{A} | \bar{B} | \bar{A} | \bar{B} | \bar{A}) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3} \times (\frac{2}{3})^5$

Similarly we can find Probability for 8th, 10th etc.

P(B Winning) = P(2nd throw) + P(4th throw) + P(6th throw) + P(8th throw) + ----

P(B Winning) $= \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times (\frac{2}{3})^3 + \frac{1}{3} \times (\frac{2}{3})^5 + \text{-------}$

This is an infinite GP with $a = \frac{1}{3}$ & $r = (\frac{2}{3})^2$

So P(B winning) $= \frac{a}{1-r} = \frac{1/3 \times 2/3}{1 - \frac{4}{9}} = \frac{2/9}{5/9} = \frac{2}{5}$